

# A Short Guide to Significant Figures

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## Quick Reference Section

Here are the basic rules for significant figures - read the full text of this guide to gain a complete understanding of what these rules really mean!

*How does one tell how many significant digits there are in a given number?*

- The leftmost digit which is not a zero is the most significant digit.
- If the number *does not* have a decimal point, the rightmost digit which is not a zero is the least significant digit.
- If the number *does* have a decimal point, the rightmost significant digit is the least significant digit, even if it's a zero.
- **Every digit between the least and most significant digits should be counted as a significant digit.**

For example, according to these rules, all of these numbers have **three significant digits**:

123  
123,000  
12.3  
 $1.23 \times 10^6$   
1.00  
0.000123

*How many significant figures should one retain in the final answer to a problem?*

- **For results obtained using addition or subtraction**, the number of places after the decimal point in the result should be less than or equal to the number of decimal places in every term in the sum.
- **For results obtained using multiplication or division**, the number of significant figures in the result should be equal to the number of significant digits in the least precise number (the number with the fewest significant digits given in the problem).

Finally, it is always good practice to give one more significant figure than is required by these rules; this helps prevent rounding errors if the number is used in later calculations.

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## Introduction

Significant figures are a shorthand way to express how certain one is about one's data and calculations coming from that data. While significant figures are by no means as precise as detailed calculations of the uncertainty of a value, they are a very useful way to estimate uncertainty quickly.

## Uncertainty and its Meaning

Any value that is the result of a scientific measurement has some uncertainty. The most precise way to state the uncertainty of a measurement is to write it as a number, plus or minus the expected error in that number. Scientists often use the standard deviation for their expected error. If you made the measurement many times, the standard deviation would be the average amount by which any given measurement differs from the mean. For example, if you measured a wire's length 30 times, and got an average length of 28.3 cm, with an average error in that length of about 0.2 cm, you would write the length as  $(28.3 \pm 0.2)$  cm if you wanted to be precise about your measurement results. This means that the majority of your measurements fall between 28.1 and 28.5 cm. (To be more specific, it means that 68% of the measurements fall between those two values, but the reasons for this are not important for the purposes of this course.)

If, for example, we then wished to find the volume of this wire, and we had a measurement of  $2.31 \pm 0.07$  mm for the wire's diameter, we would have to put these numbers into the formula for the volume of a cylinder. This would require multiplying several numbers together. While we could easily get the answer for the average volume, we need some complicated mathematics to see how the errors on the individual quantities translate through the formula to become errors on the volume. Thus it will be convenient for us to devise some simpler system of dealing with numbers that have an associated uncertainty. This system is the system of significant figures.

## Significant Figures: A Shortcut for Expressing Uncertainties

If we just want an approximate idea of the extent to which a value is certain, and we either don't want to learn the mathematical techniques, or don't want to spend the time to apply them, we can keep track of the amount of certainty of a piece of data simply by paying attention to the number of digits we use to express it. That is to say, where we choose to round off our number tells where we think uncertainty creeps in. For example, if we have a length of  $12.37 \pm 0.10$  cm, we just call the length 12.4 centimeters, to three significant figures. When we express a number with three significant figures, what we are saying is that the first two digits are essentially exactly correct, and the last one is uncertain by a small amount (generally it is only uncertain by about  $\pm 1$ ). In the example above, we rounded our answer to 12.4 cm because our answer is uncertain to  $\pm 0.1$  cm, viz., our answer is uncertain in the last digit by about 1.

## How Many Digits to Use

The question of the greatest practical importance is how many digits to include in your final answer. This is important because, as explained above, the number of digits you include in your answer shows the reader the precision of the data leading to the answer, and the accuracy of the answer. It might be useful to read this section again after reading through the following sections which explain how to determine the number of significant digits.

## Addition and Subtraction

When adding and subtracting numbers, the rules of significant figures require that *the number of places after the decimal point in the answer is less than or equal to the number of decimal places in every term in the sum*. (Treat subtraction as adding the same number with a negative sign in front of it.) If some of the numbers

have no digits after the decimal point, use the same basic rule, but don't record any digits to the right of the last digit in the least significant number. Hopefully, some examples will clarify these rules.

|           |          |          |       |
|-----------|----------|----------|-------|
| 2355.2342 | 15600.00 | 15600    | 13.7  |
| + 23.24   | + 172.49 | + 172.49 | + 1.3 |
| -----     | -----    | -----    | ----- |
| 2378.47   | 15772.49 | 15800    | 15.0  |

## Multiplication and Division

When multiplying and dividing numbers, *the number of significant digits you use is simply the same number of significant figures as is the number with the fewest significant figures*. Some examples:

|        |        |          |       |         |
|--------|--------|----------|-------|---------|
| 13.1   | 13.10  | 13.100   | 15310 | 1.00    |
| x 2.25 | x 2.25 | x 2.2500 | x 2.3 | x 10.04 |
| -----  | -----  | -----    | ----- | -----   |
| 29.5   | 29.5   | 29.475   | 35000 | 10.0    |

## Why Multiplication and Addition Have Different Rules

When you add two numbers, you add their uncertainties, more or less. If one of the numbers is smaller than the uncertainty of the other, it doesn't make much of a difference to the value (and hence, uncertainty) of the final result. Thus it is the location of the digits, not the amount of digits that is important.

When you multiply two numbers, you more or less multiply the uncertainties. Thus it is the percentage by which you are uncertain that is important -- the uncertainty in the number divided by the number itself. This is given roughly by the number of digits, regardless of their placement in terms of powers of ten. Hence the number of digits is what is important.

## Which Digits are "Significant"

In order to figure out how many significant figures to put into your final answer you must figure out how many significant figures are in each of the numbers you are working with. The rules are best explained separately for fundamental constants, physical constants, numbers not ending in 0 and numbers ending in 0.

### Fundamental Constants

Fundamental constants are numbers without units of any kind that come strictly from mathematics; they are not "measured" like most quantities in science. Some examples of these are regular integers such as 2, 10, 14, or 27; fractions such as the  $4/3$  in the formula:

$$V = \left(\frac{4}{3}\right) \pi R^3,$$

for the volume of a sphere; and constants such as  $\pi$  and  $e$ , where  $e=2.71828\dots$  is the base of the natural logarithms.

When you have one of these numbers, you should never let it determine how many significant figures you have. If the number is an integer or a fraction, just assume it has however many significant figures you need. For example, if you're multiplying  $13.75 \times 2$  where 2 is a constant in your equation, and 13.75 is a piece of data, just turn the 2 into 2.0000 and use the regular rules. If the number were 13.749878 instead of 13.75, you could

continue to add 0's after the decimal place until you had enough significant digits in 2. When you have an irrational number like  $\pi$ , look up as many digits as you need to make something else less accurate than  $\pi$ .

## Physical Constants

When you are dealing with a physical constant such as Planck's constant, the length of one light year, or the charge of an electron, you should follow the same rules as for fundamental constants. The one important difference is that since these numbers are found by experiment, and do not have any purely mathematical definition, there may be some occasions where you have a piece of data that is more certain than any available value of your physical constant. In these cases, it is acceptable to let your physical constant define your uncertainty, and hence your number of significant figures.

### Numbers Without Zeroes at the End

Numbers without zeroes at the end are the simplest case. When a given piece of data ends in a digit other than zero, all the digits in that piece of data are significant digits.

### Numbers with Zeroes at the End

Numbers ending in zero are more complicated. In these cases, you must determine whether the zeroes are significant digits or just place holders. Zeroes between the last non-zero digit and the decimal point may or may not be significant. If the decimal point is shown, then these zeroes are significant. If the decimal point is not shown, these zeroes are said to be indeterminate since it is not clear whether they are or are not significant digits. Zeroes after a decimal point are always significant digits, because without them the actual value of the number is no different, so we assume they are placed there to show additional certainty in the value of the number.

Examples:

- 130. has three significant digits.
- 130 is indeterminate and could have either two or three significant digits.
- 130.0 has four significant digits.
- 1.000000 has seven significant digits.
- 100000. has six significant digits.
- 100000 is indeterminate and could have one, two, three, four, five or six significant digits.
- $1.30 \times 10^2$  is the way you write 130 if you want to make it clear that there are **3**, **not 2** significant digits in the number.

## How Strict are these Rules? Are there Exceptions?

In a way, yes. One is that you should always carry through all the digits you have available in a calculation until the very end, where you can then truncate your answer to the correct number of significant digits. In other words, during some intermediate step of the calculation, don't attempt to eliminate the final few digits on your calculator or write down an intermediate answer with fewer digits and then re-type that new number into the calculator. By the guidelines given above, you might be tempted to round off midway through but doing so actually introduces a small (and sometimes non-negligible) amount of "truncation error" which is especially prone to happen if you have a complicated calculation involving many steps. This type of error is totally avoidable (as opposed to measurement errors, which we are

stuck with) and therefore it is encouraged and even more correct to keep all those intermediate digits until the final answer.

As a corollary of the above statement, it is often desirable to add one more significant digit in your final answer than the rules we presented above would otherwise have you do. One example where this is desirable is if your result was to, in turn, be plugged into a future calculation where "truncation errors" could become important. Under what kind of situations this becomes important is beyond the scope of this short guide.

You should bear in mind the significant figure rules when doing calculations on the exams. There will be grade penalties for ignoraning these rules. A particularly glaring illustration would be the case where your calculations involve division of 123 by 45. A calculator gives the answer 2.7333333 but the correct answer using the significant digit rules is 2.7; students who give the 2.7333333 answer will lose points.